647510 **BSc5MatC501x** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (CBCS) Examination
March/April-2026 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 2:30 Hours **Marks:70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any TWO. (10)

(1) State and prove the Hausdorff principle for metric spaces.

(2) Define a metric space. Let (X, d) be a metric space, show that closure of every subset of X is closed in X.

(3) Define Cantor set on [0, 1]. Show that a cantor set is closed. Comment on the existence of minimum of Cantor set and if possible find it.

Que.1(B) Answer any ONE. (04)

(1) (a) Prove or disprove: $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

(b) Is the set of rational numbers dense in the set of real numbers? Justify.

(2) (a) Check whether $P = (2, \infty) \cup \{-1\}$ is open or close and justify.

(b) Define boundary point. Find boundary points of set $P = (2, \infty) \cup \{-1\}$.

Que.2(A) Answer any TWO. (10)

(1) Prove that any bounded function f defined on [a, b] is R-integrable if and only if for every $\in > 0$ there exist a partition P of [a, b] such that $U(P, f) - L(P, f) < \in$.

(2) If functions f and g are R-integrable on [a, b] then show that $f + g$ is also R-integrable on [a, b].

(3) For $0 \le x \le \dfrac{\pi}{2}$, show that $f(x) = \sin x$ is R-integrable and find $\displaystyle\int_0^{\frac{\pi}{2}} \sin x dx$.

Que.2(B) Answer any ONE. (04)

(1) For $f(x) = \dfrac{1}{1+x}$, $x \in [0,1]$ and partition $P = \left\{0, \dfrac{2}{7}, \dfrac{3}{7}, \dfrac{6}{7}, \dfrac{13}{14}, 1\right\}$, evaluate $U(P, f)$ and $L(P, f)$.

(2) Solve $\displaystyle\int_0^1 (2x - 3)dx$ using the definition of Riemann integration.

Que.3(A) Answer any TWO. (10)

(1) Show that $\displaystyle\int_0^{\frac{\pi}{4}} x \tan x \, dx \le \frac{\pi}{8} \log_e 2$.

(2) State and prove the first mean value theorem for Riemann integration.

(3) Define a group. Show that $G = \mathbb{R} - \{-1\}$ is a group under $*$ operation, defined by $a * b = a + b + ab \ \forall a, b \in G$.

Que.3(B) Answer any ONE. (04)

(1) Evaluate : $\displaystyle\lim_{n \to \infty} \frac{1}{n} \left[e^{\frac{3}{n}} + e^{\frac{6}{n}} + \dots + e^{\frac{3n}{n}} \right]$.

(2) Prove that the set of cube roots of unity form a commutative group with respect to the binary operation of multiplication of complex numbers.

Que.4(A) Answer any TWO. (10)

(1) State and prove Lagrange's theorem for a subgroup of a finite group.

(2) State and prove Euler's theorem.

(3) Define a symmetric group and alternating group. Prepare a group table for alternating group A_3 of symmetric group S_3 in usual notations.

Que.4(B) Answer any ONE. (04)

(1) Let G be a finite group of even order. Prove that there exist an element $a \neq e$ in G such that $a = a^{-1}$.

(2) In a finite group G, prove that $o(bab^{-1}) = o(a)\ \forall a, b \in G$.

Que.5(A) Answer any TWO. (10)

(1) Prove that an infinite cyclic group has exactly two generators.

(2) Prove that the relation $\cong$ of isomorphism of groups is an equivalence relation.

(3) State and prove Cayley's theorem.

Que.5(B) Answer any ONE. (04)

(1) Show that $(\mathbb{R}^{+}, \cdot) \cong (\mathbb{R}, +)$

(2) Let G be a group and H be its subgroup. If index of H in G is 2 then prove that H is a normal subgroup of G.

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